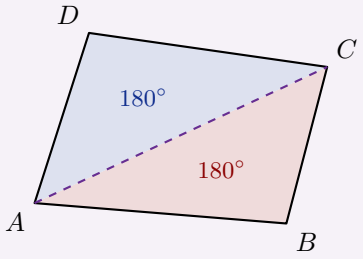
Patrulatere

Teorie pe desene · ce înseamnă fiecare marcaj: — *laturi egale* → *laturi paralele* ⌐ *unghi drept*

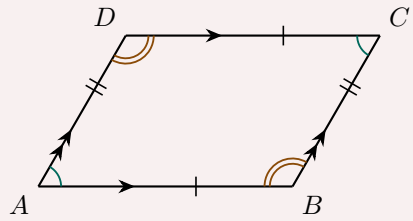
Patrulaterul convex



- 4 vârfuri, 4 laturi, 2 diagonale (AC, BD).
- Diagonala AC îl taie în 2 triunghiuri, deci suma unghiurilor e $2 \cdot 180°$:
  $\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$
- $P = AB + BC + CD + DA$

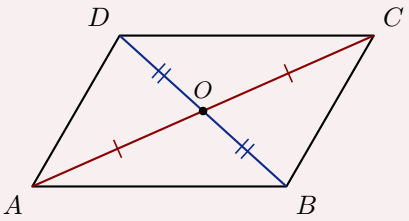
Ex.: $\hat{A} = 80°$, $\hat{B} = 100°$, $\hat{C} = 70°$ $\Rightarrow \hat{D} = 110°$.

Paralelogramul · laturile opuse sunt paralele



laturi și unghiuri

- $AB \parallel CD$, $AD \parallel BC$ (definiția)
- laturile opuse congruente:
  $[AB] \equiv [CD]$, $[AD] \equiv [BC]$
- unghiurile opuse congruente:
  $\sphericalangle A \equiv \sphericalangle C$, $\sphericalangle B \equiv \sphericalangle D$

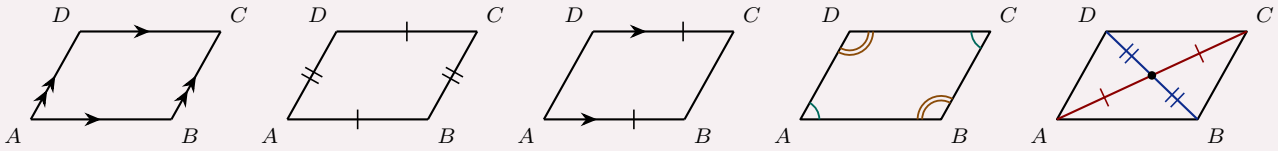


diagonalele

- unghiurile alăturate sunt suplementare: $\hat{A} + \hat{B} = 180°$
- diagonalele se înjumătățesc:
  $[AO] \equiv [OC]$, $[BO] \equiv [OD]$

Ex.: $\hat{A} = 70° \Rightarrow \hat{C} = 70°$ și $\hat{B} = \hat{D} = 180° - 70° = 110°$.

Ca să arăt că patrulaterul convex $ABCD$ e paralelogram, e suficientă UNA dintre:



1. laturile opuse paralele (2 câte 2)

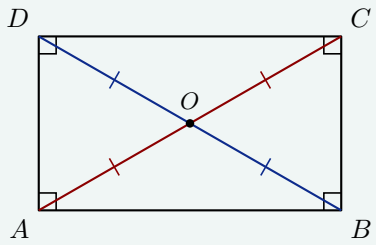
2. laturile opuse congruente (2 câte 2)

3. două laturi opuse paralele *și* congruente

4. unghiurile opuse congruente

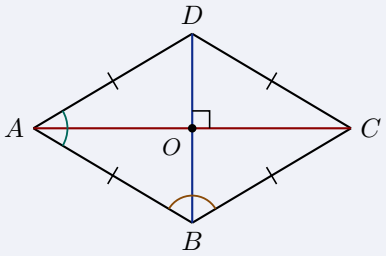
5. diagonalele se înjumătățesc

Dreptunghiul



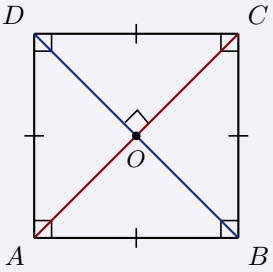
- = paralelogram cu **un unghi drept**
- toate cele 4 unghiuri au 90°
- diagonalele sunt **congruente**: $[AC] \equiv [BD]$
- $OA = OB = OC = OD$
- paralelogram cu diagonale congruente $\Rightarrow$ dreptunghi

Rombul



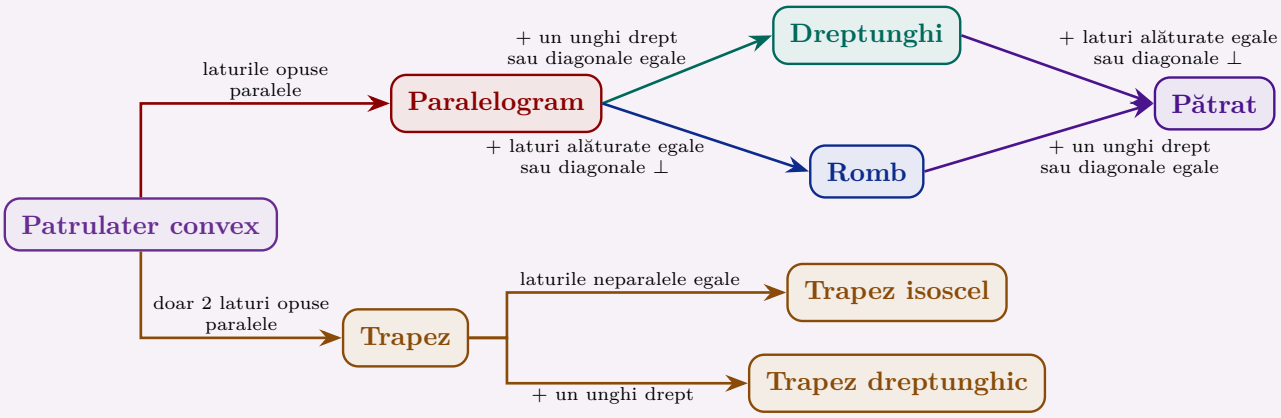
- = paralelogram cu **2 laturi alăturate congruente**
- toate laturile congruente
- diagonale **perpendiculare**: $AC \perp BD$
- diagonalele = bisectoarele unghiurilor
- paralelogram cu diagonale perpendiculare $\Rightarrow$ romb

Pătratul



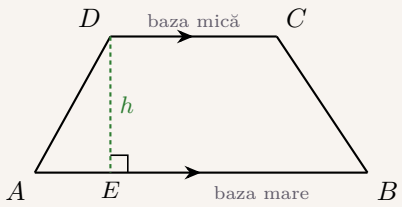
- e **și dreptunghi, și romb**
- 4 laturi egale, 4 unghiuri drepte
- diagonalele: congruente, perpendiculare, se înjumătățesc
- diagonala face 45° cu laturile
- diagonala $d = l\sqrt{2}$ (Pitagora)

Cine ce este? Pe săgeată: ce condiție adaug

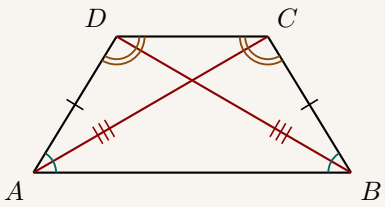


Orice pătrat e și dreptunghi, și romb. Orice dreptunghi și orice romb e paralelogram, deci are *toate* proprietățile paralelogramului. Trapezul **nu** e paralelogram. (Pe desene, „egale” = congruente.)

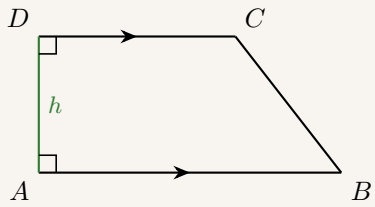
Trapezul · doar două laturi opuse sunt paralele



- bazele: $AB \parallel CD$
- înălțimea: $h = DE \perp AB$
- $\hat{A} + \hat{D} = 180°$, $\hat{B} + \hat{C} = 180°$

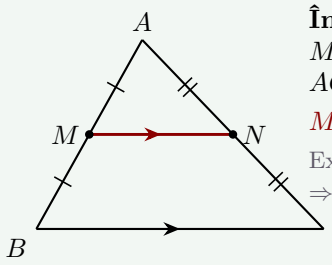


- **isoscel**: $[AD] \equiv [BC]$
- unghiuri egale la bază: $\sphericalangle A \equiv \sphericalangle B$, $\sphericalangle C \equiv \sphericalangle D$
- diagonale: $[AC] \equiv [BD]$



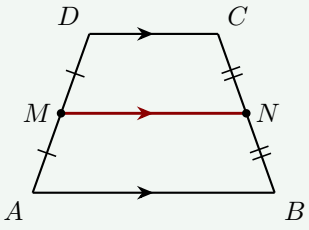
- **dreptunghic**: $AD \perp$ baze
- $\hat{A} = \hat{D} = 90°$
- latura AD este chiar înălțimea

Linia mijlocie · unește mijloacele a două laturi



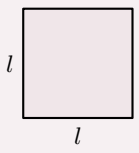
În triunghi
M, N mijloacele lui AB, $AC \Rightarrow$
$MN \parallel BC$ și $MN = \dfrac{BC}{2}$
Ex.: $BC = 10$ cm
$\Rightarrow MN = 5$ cm.

Și invers: paralela prin mijlocul unei laturi la a doua latură trece prin mijlocul celei de-a treia.

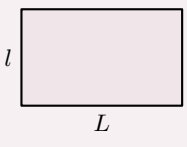


În trapez
M, N mijloacele laturilor neparalele $\Rightarrow$
$MN \parallel AB \parallel CD$
$MN = \dfrac{B + b}{2}$
Ex.: $B = 12$ cm, $b = 6$ cm
$\Rightarrow MN = 9$ cm.

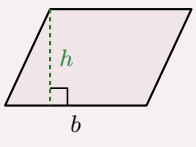
Ariile (punctat = înălțimea; în exemple, lungimile sunt în cm)



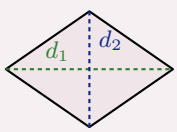
Pătrat
$\mathcal{A} = l^2$
$l = 5$
$\Rightarrow \mathcal{A} = 25$ cm^2



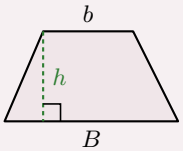
Dreptunghi
$\mathcal{A} = L \cdot l$
$L = 6$, $l = 4$
$\Rightarrow \mathcal{A} = 24$ cm^2



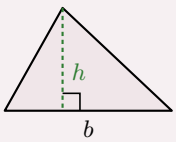
Paralelogram
$\mathcal{A} = b \cdot h$
$b = 7$, $h = 4$
$\Rightarrow \mathcal{A} = 28$ cm^2



Romb
$\mathcal{A} = \dfrac{d_1 \cdot d_2}{2}$
$d_1 = 8$, $d_2 = 6$
$\Rightarrow \mathcal{A} = 24$ cm^2



Trapez
$\mathcal{A} = \dfrac{(B + b) \cdot h}{2}$
$B = 10$, $b = 6$, $h = 4$
$\Rightarrow \mathcal{A} = 32$ cm^2



Triunghi
$\mathcal{A} = \dfrac{b \cdot h}{2}$
$b = 8$, $h = 5$
$\Rightarrow \mathcal{A} = 20$ cm^2